

# Package: PEIP (via r-universe)

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**Description** Several functions introduced in Aster et al.'s book on inverse theory. The functions are often translations of MATLAB code developed by the authors to illustrate concepts of inverse theory as applied to geophysics. Generalized inversion, tomographic inversion algorithms (conjugate gradients, 'ART' and 'SIRT'), non-linear least squares, first and second order Tikhonov regularization, roughness constraints, and procedures for estimating smoothing parameters are included.

**License** GPL (>= 2)

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PEIP-package

*Inverse Theory Functions for PEIP book*

---

### Description

Auxilliary functions and routines for running the examples and excersizes described in the book on inverse theory.

### Details

These functions are used in conjunction with the example described in the PEIP book.

There is one C-code routine, interp2grid. This is introduced to replicate the MATLAB code interp2. It does not work exactly as the matlab code prescribes.

In the PEIP library one LAPACK routine is called: dggsvd. In R, LAPACK routines are stored in slightly different locations on Linux, Windows and Mac computers. Be aware. This will come up in examples from Chapter 4.

Almost all examples work as scripts run with virtually no user input, e.g.

### Author(s)

Jonathan M. Lees<jonathan.lees.edu> Maintainer:Jonathan M. Lees<jonathan.lees.edu>

### References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

---

Ainv

*An Inverse Solution*

---

### Description

QR decomposition solution to  $Ax=b$

### Usage

```
Ainv(GAB, x, tol = 1e-12)
```

### Arguments

GAB	design matrix
x	right hand side
tol	tolerance for singularity

**Details**

I needed something to make up for the lame-o matlab code that does this  $h = G \setminus x$  to get the inverse

**Value**

Inverse Solution

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```
set.seed(2015)
GAB = matrix(runif(36), ncol=6)
truex = rnorm(ncol(GAB))
rhs = GAB %*% truex

rhs = as.vector(rhs )

tout = Ainv(GAB, rhs, tol = 1e-12)
tout - truex
```

---

art

*ART Inverse solution*

---

**Description**

ART algorithm for solving sparse linear inverse problems

**Usage**

```
art(A, b, tolx, maxiter)
```

**Arguments**

A	Constraint matrix
b	right hand side
tolx	difference tolerance for successive iterations (stopping criteria)
maxiter	maximum iterations (stopping criteria).

**Details**

Alpha is a damping factor. If  $\alpha < 1$ , then we won't take full steps in the ART direction. Using a smaller value of alpha (say  $\alpha = .75$ ) can help with convergence on some problems.

**Value**

x                    solution

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
set.seed(2015)
G = setDesignG()
### % Setup the true model.
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);

mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;
mtruem[10,9]=1; mtruem[10,11]=1;
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;
mtruem[2,3]=1; mtruem[2,4]=1;
mtruem[3,3]=1; mtruem[3,4]=1;

### % reshape the true model to be a vector
mtruev=as.vector(mtruem);

### % Compute the data.
dtrue=G %*% mtruev;

### % Add the noise.

d=dtrue+0.01*rnorm(length(dtrue));

mkac<-art(G,d,0.01,200)
par(mfrow=c(1,2))
imagesc(matrix(mtruem,16,16) , asp=1 , main="True Model" );

imagesc(matrix(mkac,16,16) , asp=1 , main="ART Solution" );
```

**Description**

Bartlett (triangle) window of length m

**Usage**

bartl(m)

**Arguments**

m                    integer, length of vector

**Value**

vector

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

bartl(11)

---

bayes

*Bayes Inversion*

---

**Description**

Given a linear inverse problem  $Gm=d$ , a prior mean mprior and covariance matrix covm, data d, and data covariance matrix covd, this function computes the MAP solution and the corresponding covariance matrix.

**Usage**

bayes(G, mprior, covm, d, covd)

**Arguments**

G	Design Matrix
mprior	vector, prior model
covm	vector, model covariance
d	vector, right hand side
covd	vector, data covariance

**Value**

vector model

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
## Not run:
set.seed(2015)
G = setDesignG()
###
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);

mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;
mtruem[10,9]=1; mtruem[10,11]=1;
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;
mtruem[2,3]=1; mtruem[2,4]=1;
mtruem[3,3]=1; mtruem[3,4]=1;

###
mtruev=as.vector(mtruem);
imagesc(matrix(mtruem,16,16) , asp=1 , main="True Model" );

matrix(mtruem,16,16) , asp=1 , main="True Model" )

###
dtrue=G %%% mtruev;

###
d=dtrue+0.01*rnorm(length(dtrue));
covd = 0.1*diag( nrow=length(d) )
covm = 1*diag( nrow=dim(G)[2] )

## End(Not run)
```

---

blf2

*Bounded least squares*


---

**Description**

Bounded least squares

**Usage**

```
blf2(A, b, c, delta, l, u)
```

**Arguments**

A	Design Matrix
b	Right hand side
c	matrix weight on x
delta	tolerance
l	lower bound
u	upper bound

**Details**

Solves the problem:  $\min/\max c'x$  where  $\|Ax-b\| \leq \delta$  and  $l \leq x \leq u$ .

**Value**

x	solution
---	----------

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Stark, P.B. , and R. L. Parker, *Bounded-Variable Least-Squares: An Algorithm and Applications*, Computational Statistics 10:129-141, 1995.

**Examples**

```
### set up an inverse problem:Shaw problem

n = 20
G = shawG(n,n)

spike = rep(0,n)
spike[10] = 1

spiken = G %*% spike

wts = rep(1, n)
delta = 1e-03
set.seed(2015)
dspiken = spiken + 6e-6 *rnorm(length(spiken))

lb = spike - (.2) * wts
```



```
ub = spike + (.2) * wts
dspiken = dspiken
blf2(G, dspiken, wts , delta, lb, ub)
```

---

cgl<sub>s</sub>

*Conjugate gradient Least squares*

---

### Description

Conjugate gradient Least squares

### Usage

```
cgls(Gmat, dee, niter)
```

### Arguments

Gmat	input matrix
dee	right hand side
niter	max number of iterations

### Details

Performs niter iterations of the CGLS algorithm on the least squares problem  $\min \text{norm}(G^*m-d)$ . Gmat should be a sparse matrix.

### Value

X	matrix of models
rho	misfit norms
eta	model norms

### Author(s)

Jonathan M. Lees<[jonathan.lees@unc.edu](mailto:jonathan.lees@unc.edu)>

### References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
set.seed(11)
#### perfect data with no noise
n <- 5
A <- matrix(runif(n*n),nrow=n)
B <- runif(n)
### get right-hand-side (data)
trhs = as.vector( A %*% B )
Lout = cglS(A, trhs , 15)

### solution is
Lout$X[,15]

Lout$X[,15] - B
```

---

chi

*Chi function*

---

**Description**

Chi function

**Usage**

```
chi(x, n)
```

**Arguments**

x	value
n	degrees of freedom

**Value**

function evaluated

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
x = seq(0, 10, length=100)
n = 5
y=chi(x, n)
plot(x, y)
```

---

chi2cdf

*Chi-Sq CDF*

---

**Description**

Computes the Chi<sup>2</sup> CDF, using a transformation to N(0,1) on page 333 of Thistead, Elements of Statistical Computing.

**Usage**

```
chi2cdf(x, n)
```

**Arguments**

x	end value of chi <sup>2</sup> pdf to integrate to. (scalar)
n	degrees of freedom (scalar)

**Details**

Note that x and m must be scalars.

**Value**

p	probability that Chi <sup>2</sup> random variable is less than or equal to x (scalar).
---	--

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
x= seq(from=0.1, to=0.9, length=20)
chi2cdf(x , 3)
```

---

`chi2inv`*Inverse Chi-Sq*

---

**Description**

Inverse Chi-Sq

**Usage**`chi2inv(x, n)`**Arguments**

`x` probability that Chi<sup>2</sup> random variable is less than or equal to `x` (scalar).  
`n` degrees of freedom (scalar)

**Details**

Computes the inverse Chi<sup>2</sup> distribution corresponding to a given probability that a Chi<sup>2</sup> random variable with the given degrees of freedom is less than or equal to `x`. Uses `chi2cdf.m`.

**Value**

corresponding value of `x` for given probability.

**Author(s)**

Jonathan M. Lees<[jonathan.lees@unc.edu](mailto:jonathan.lees@unc.edu)>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**See Also**

`chi`, `chi2cdf`

**Examples**

```
x = seq(from=0.1, to=0.9, length=10)
h = chi2cdf(x, 3)

chi2inv(h, 3)
```

---

dcost	<i>cosine transform</i>
-------	-------------------------

---

**Description**

Computes the column-by-column discrete cosine transform of X.

**Usage**

```
dcost(X)
```

**Arguments**

X	Time series matrix
---	--------------------

**Value**

cosine transformed data

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```
x <- 1:4  
  
### compare fft with cosine transform  
fft(x)  
  
dcost(x)
```

---

error.bar	<i>Plot Error Bar</i>
-----------	-----------------------

---

**Description**

Plot Error Bar

**Usage**

```
error.bar(x, y, lo, hi, pch = 1, col = 1, barw = 0.1, add = FALSE, ...)
```

**Arguments**

x	X-values
y	Y-values
lo	Lower limit of error bars
hi	Upper limit of error bars
pch	plotting character
col	color
barw	width of the bar
add	logical, add=FALSE starts a new plot
...	other plotting parameters

**Value**

graphical side effects

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```
x = 1:10
y = 2*x+5
zup = rnorm(10)
```

```
zup = zup-min(zup)+.5
zdown = rnorm(10)
zdown = zdown-min(zdown)+.2
```

```
#### example with same error on either side:
error.bar(x, y, y-zup, y+zup, pch = 1, col = 'brown' , barw = 0.1, add =
FALSE)
```

```
#### example with different error on either side:
error.bar(x, y, y-zdown, y+zup, pch = 1, col = 'brown' , barw = 0.1, add
= FALSE)
```

---

`flipGSVD`*Flip output of GSVD*

---

**Description**

Flip (reverse order) output of GSVD

**Usage**

```
flipGSVD(vs, d1 = c(50, 50), d2 = c(48, 50))
```

**Arguments**

<code>vs</code>	list output of GSVD
<code>d1</code>	dimensionals of A
<code>d2</code>	dimensions of B

**Details**

This flipping of the matrix is done to agree with the Matlab code.

**Value**

Same as GSVD, but order of eigenvectors is reversed.

<code>U</code>	m by m orthogonal matrix
<code>V</code>	p by p orthogonal matrix, $p=\text{rank}(B)$
<code>X</code>	n by n nonsingular matrix
<code>C</code>	singular values, m by n matrix with diagonal elements shifted from main diagonal
<code>S</code>	singular values, p by n diagonal matrix

**Note**

The GSVD routines are from LAPACK.

**Author(s)**

Jonathan M. Lees<[jonathan.lees@unc.edu](mailto:jonathan.lees@unc.edu)>

**See Also**

GSVD

**Examples**

```

set.seed(12)

n <- 5
A <- matrix(runif(n*n),nrow=n)
B <- matrix(runif(n*n),nrow=n)

VS = GSVD(A, B)

FVS = flipGSVD(VS, d1 = dim(A) , d2 = dim(B) )
## see that order of eigen vectors is reversed
diag(VS$S)
diag(FVS$S)

```

---

gcval

*Get c-val*


---

**Description**

Extract the smallest regularization parameter.

**Usage**

```
gcval(U, s, b, npoints)
```

**Arguments**

U	U matrix from gsvd(G, L)
s	[diag(C) diag(S)] which are the lambdas and mus from the gsvd
b	the data to try and match
npoints	number of alphas to estimate

**Details**

Evaluate the GCV function `gcv_function` at `npoints` points.

**Value**

List:

reg_min	alpha with the minimal g (scalar)
g	$\ Gm_{\text{alpha},L} - d\ ^2 / (\text{Tr}(I - GG\#))^2$
alpha	alpha for the corresponding g

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>



**See Also**

gcv\_function

**Examples**

```

set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );

BIGU = flipGSVD(littleU, dim(G), dim(L1) )

U1 = BIGU$U
V1 =BIGU$V
X1=BIGU$X
Lam1=BIGU$C
M1=BIGU$S

lam=sqrt(diag(t(Lam1 %% Lam1)));

mu=sqrt(diag(t(M1)*%M1));

p=rank(L1);

sm1=cbind(lam[1:p],mu[1:p])

### % get the gcv values varying alpha

###
ngcvpoints=1000;

HI = gcvval(U1,sm1,t,ngcvpoints);

```

gcv\_function

*gcv\_func***Description**

Auxiliary routine for GCV calculations

**Usage**

```
gcv_function(alpha, gamma2, beta)
```

**Arguments**

alpha	parameter
gamma2	square of the gamma from the gsvd
beta	projected data to fit

**Value**

vector,  $g - \|Gm_{\alpha,L} - d\|^2 / (\text{Tr}(I - GG\#)^2)$

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

---

get\_1\_rough

*One-D Roughening*

---

**Description**

Returns a 1D differentiating matrix operating on a series with n points.

**Usage**

```
get_1_rough(n, deg)
```

**Arguments**

n	integer, number of data points
deg	order of the derivative to approximate

**Details**

Used to get first and 2nd order roughening matrices for 1-D problems

**Value**

Matrix:discrete differentiation matrix

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
### first order roughening matrix for a 10 by 10 model: a sparse matrix
N = 10
L1 = get_l_rough(10,1);

### second order roughening matrix for a 10 by 10 model
N = 10
L2 = get_l_rough(10,2);
```

---

ginv

*Get inverse*

---

**Description**

Get inverse of matrix or solve  $Ax=b$ .

**Usage**

```
ginv(G, x, tol = 1e-12)
```

**Arguments**

G	Design Matrix
x	right hand side
tol	tolerance

**Details**

This function used as alternative to matlab code that does this  $h = G \setminus x$  to get the inverse

**Value**

inverse as a N by 1 matrix.

**Note**

Be careful about the usage of tolerance

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**See Also**

solve, Ainv

**Examples**

```
set.seed(2015)
GAB = matrix(runif(36), ncol=6)
truex = rnorm(ncol(GAB))
rhs = GAB %*% truex

rhs = as.vector(rhs )

tout = ginv(GAB, rhs, tol = 1e-12)
tout - truex
```

---

GSVD

*Generalized SVD*

---

**Description**

Wrapper for generalized svd from LAPACK

**Usage**

GSVD(A, B)

**Arguments**

A	Matrix, see below
B	Matrix, see below

**Details**

The A and B matrices will be,  $A=U^*C^*t(X)$  and  $B=V^*S^*t(X)$ , respectively.

Since PEIP is based on a book, which is itself based on MATLAB routines, the convention here follows the book. The R implementation uses LAPACK and wraps the function so the output will comply with the book. See page 104 of the second edition of the Aster book cited below. That said, the purpose is to find an inversion of the form  $Y = t(A \ aB)$ , where  $a$  is a regularization parameter,  $B$  is smoothing matrix and  $A$  is the design matrix for the forward problem. The input matrices  $A$  and  $B$  are assumed to have full rank, and  $p = \text{rank}(B)$ . The generalized singular values are then  $\gamma = \lambda/\mu$ , where  $\lambda = \sqrt{\text{diag}(t(C)^*C)}$  and  $\mu = \sqrt{\text{diag}(t(S)^*S)}$ .

**Value**

U	m by m orthogonal matrix
V	p by p orthogonal matrix, $p=\text{rank}(B)$
X	n by n nonsingular matrix
C	singular values, m by n matrix with diagonal elements shifted from main diagonal
S	singular values, p by n diagonal matrix

**Note**

Requires R version of LAPACK. The code is a wrapper for the dggsvd function in LAPACK. The author thanks Berend Hasselman for advice and help preparing this function.

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**See Also**

flipGSVD

**Examples**

```
# Example from NAG F08VAF

A <- matrix(1:15, nrow=5, ncol=3)
B <- matrix(c(8,1,6,
              3,5,7,
              4,9,2), nrow=3, byrow=TRUE)

z <- GSVD(A,B)
C <- z$C
S <- z$S
sqrt(diag(t(C) %*% C)) / sqrt(diag(t(S) %*% S))
testA = A - z$U %*% C %*% t(z$X)
testB = B - z$V %*% S %*% t(z$X)

print(testA)
print(testB)
```

---

`idcost`*Inverse cosine transform*

---

**Description**

Takes the column-by-column inverse discrete cosine transform of Y.

**Usage**

```
idcost(Y)
```

**Arguments**

Y                    Input cosine transform

**Value**

Time series

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**See Also**

`dcost`

**Examples**

```
x <- 1:4

### compare fft with cosine transform
fft(x)

zig = dcost(x)
zag = idcost(zig)
```

---

`imagesc`*Image Display*

---

**Description**

Display image in matlab format, i.e. flip and transpose.

**Usage**

```
imagesc(G, col = grey((1:99)/100), ...)  
contoursc(G, ...)
```

**Arguments**

<code>G</code>	Image matrix
<code>col</code>	color scale
<code>...</code>	graphical parameters

**Details**

Program flips image and transposes prior to plotting. The contour version does the same and can be used to add contours.

**Value**

graphical side effects

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);  
  
mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;  
mtruem[10,9]=1; mtruem[10,11]=1;  
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;  
mtruem[2,3]=1; mtruem[2,4]=1;  
mtruem[3,3]=1; mtruem[3,4]=1;  
  
imagesc(mtruem, asp=1)
```

interp2grid

*Bilinear and Bicubic Interpolation to Grid***Description**

This code includes a bicubic interpolation and a bilinear interpolation adapted from Numerical Recipes in C: The art of scientific computing (chapter 3... bicubic interpolation) and a bicubic interpolation from in java code.

Inputs are a list of points to interpolate to and from raster objects of class 'asc' (adehabitat package), 'RasterLayer' (raster package) or 'SpatialGridDataFrame' (sp package).

**Usage**

```
interp2grid(mat,xout,yout,xin=NULL,yin=NULL,type=2)
```

**Arguments**

mat	a matrix of data that can be a raster matrix of class 'asc' (adehabitat package), 'RasterLayer' (raster package) or 'SpatialGridDataFrame' (sp package) NA values are not permitted.. data must be complete.
xout	a vector of data representing x coordinates of the output grid. Resulting grid must have square cell sizes if mat is of class 'asc', 'RasterLayer' or 'SpatialGridDataFrame'.
yout	a vector of data representing x coordinates of the output grid. Resulting grid must have square cell sizes if mat is of class 'asc', 'RasterLayer' or 'SpatialGridDataFrame'.
xin	a vector identifying the locations of the columns of the input data matrix. These are automatically populated if mat is of class 'asc', 'RasterLayer' or 'SpatialGridDataFrame'.
yin	a vector identifying the locations of the rows of the input data matrix. These are automatically populated if mat is of class 'asc', 'RasterLayer' or 'SpatialGridDataFrame'.
type	an integer value representing the type of interpolation method used. 1 - bilinear adapted from Numerical Recipes in C 2 - bicubic adapted from Numerical Recipes in C 3 - bicubic adapted from online java code

**Value**

Returns a matrix of the originating class.

**Author(s)**

Jeremy VanDerWal <jjvanderwal@gmail.com>



**Examples**

```

tx = seq(0,3,0.1)
ty = seq(0,3,0.1)

tmat = matrix(runif(16,1,16),nrow=4)
txin = seq(0,3,length=4)
tyin = seq(0,3,length=4)

bilinear1 = interp2grid(tmat,tx,ty,txin, tyin, type=1)
bicubic2 = interp2grid(tmat,tx,ty,txin, tyin, type=2)
bicubic3 = interp2grid(tmat,tx,ty,txin, tyin, type=3)

par(mfrow=c(2,2),cex=1)
image(tmat,main='base',zlim=c(0,16),col=heat.colors(100))
image(bilinear1,main='bilinear',zlim=c(0,16),col=heat.colors(100))
image(bicubic2,main='bicubic2',zlim=c(0,16),col=heat.colors(100))
image(bicubic3,main='bicubic3',zlim=c(0,16),col=heat.colors(100))

```

---

 irls

*Iteratively reweight least squares*


---

**Description**

Uses the iteratively reweight least squares strategy to find an approximate  $L_p$  solution to  $Ax=b$ .

**Usage**

```
irls(A, b, tolR, tolX, p, maxiter)
```

**Arguments**

A	Matrix of the system of equations.
b	Right hand side of the system of equations
tolR	Tolerance below which residuals are ignored
tolX	Stopping tolerance. Stop when $(\text{norm}(\text{newx}-x))/(1+\text{norm}(x)) < \text{tolx}$
p	Specifies which p-norm to use (most often, $p=1$ .)
maxiter	Limit on number of iterations of IRLS

**Details**

Use to get L-1 norm solution of inverse problems.

**Value**

x	Approximate $L_p$ solution
---	----------------------------

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
t = 1:10
y=c(109.3827,187.5385,267.5319,331.8753,386.0535,
428.4271,452.1644,498.1461,512.3499,512.9753)
sigma = rep(8, length(y))
N=length(t);

### % Introduce the outlier
y[4]=y[4]-200;

G = cbind( rep(1, N), t, -1/2*t^2 )

### % Apply the weighting

yw = y/sigma;

Gw = G/sigma

m2 = solve( t(Gw) %*% Gw , t(Gw) %*% yw, tol=1e-12 )

### Solve for the 1-norm solution

m1 = irls(Gw,yw,1.0e-5,1.0e-5,1,25)
m1
```

---

 irlsl1reg

*L1 least squares with sparsity*


---

**Description**

Solves the system  $Gm=d$  using sparsity regularization on  $Lm$ . Solves the L1 regularized least squares problem:  $\min \text{norm}(G*m-d,2)^2 + \alpha * \text{norm}(L*m,1)$

**Usage**

```
irlsl1reg(G, d, L, alpha, maxiter = 100, tolx = 1e-04, tolr = 1e-06)
```

**Arguments**

G	design matrix
d	right hand side
L	regularization matrix
alpha	regularization parameter
maxiter	Maximum number of IRLS iterations
tolx	Tolerance on successive iterates
tolr	Tolerance below which we consider an element of $L^*m$ to be effectively zero

**Value**

m	model vector
---	--------------

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```

n = 20
G = shawG(n,n)

spike = rep(0,n)
spike[10] = 1

spiken = G %*% spike

wts = rep(1, n)
delta = 1e-03
set.seed(2015)
dspiken = spiken + 6e-6 *rnorm(length(spiken))
L1 = get_l_rough(n,1);
alpha = 0.001

k = irlsl1reg(G, dspiken, L1, alpha, maxiter = 100, tolx = 1e-04, tolr = 1e-06)

plotconst(k,-pi/2,pi/2, ylim=c(-.2, 0.5), xlab="theta", ylab="Intensity" );

```

---

kac	<i>Kaczmarz</i>
-----	-----------------

---

**Description**

Implements Kaczmarz's algorithm to solve a system of equations iteratively

**Usage**

```
kac(A, b, tolX, maxiter)
```

**Arguments**

A	Constraint matrix
b	right hand side
tolX	difference tolerance for successive iterations (stopping criteria)
maxiter	maximum iterations (stopping criteria)

**Value**

x	solution
---	----------

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
set.seed(2015)
G = setDesignG()
### % Setup the true model.
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);

mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;
mtruem[10,9]=1; mtruem[10,11]=1;
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;
mtruem[2,3]=1; mtruem[2,4]=1;
mtruem[3,3]=1; mtruem[3,4]=1;

### % reshape the true model to be a vector
mtruev=as.vector(mtruem);

### % Compute the data.
```

```
dtrue=G %**% mtruev;  
### % Add the noise.  
  
d=dtrue+0.1*rnorm(length(dtrue));  
  
mkac<-kac(G,d,0.0,200)  
par(mfrow=c(1,2))  
imagesc(matrix(mtruev,16,16) , asp=1 , main="True Model" );  
  
imagesc(matrix(mkac,16,16) , asp=1 , main="Kacz Solution" );
```

---

linesconst

*Plot constant model*

---

### Description

Add to plotting model in piecewise constant form over  $n$  subintervals, where  $n$  is the length of  $x$ .

### Usage

```
linesconst(x, l, r, ...)
```

### Arguments

<code>x</code>	model to be plotted
<code>l</code>	left endpoint of plot
<code>r</code>	right endpoint of plot
<code>...</code>	graphical parameters

### Details

Used for plotting vector models

### Value

graphical side effects

### Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

### See Also

plotconst

**Examples**

```
zip = runif(25)
plotconst(zip, 0, 1 )
linesconst(runif(25) , 0, 1 , col='red' )
```

---

lmarq

*Lev-Marquardt Inversion*


---

**Description**

Use the Levenberg-Marquardt algorithm to minimize  $f(p)=\sum(F_i(p)^2)$

**Usage**

```
lmarq(afun, ajac, p0, tol, maxiter)
```

**Arguments**

afun	name of the function F(x)
ajac	name of the Jacobian function J(x)
p0	initial guess
tol	stopping tolerance
maxiter	maximum number of iterations allowed

**Value**

pstar	best solution found.
iter	Iteration count.

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```
fun<-function(p){
### Compute the function values.
fvec=rep(0,length(TM))
fvec=(Q*exp(-D^2*p[1]/(4*p[2]*TM))/(4*pi*p[2]*TM) - H)/SIGMA
return(fvec)
}
jac <-function( p)
{
### use known formula for the derivatives in the Jacobian
n=length(TM)
J= matrix(0,nrow=n,ncol=2)
```

```

    J[,1]=(-Q*D^2*exp(-D^2*p[1]/(4*p[2]*TM))/(16*pi*p[2]^2*TM^2))/SIGMA

    J[,2]=(Q/(4*pi*p[2]^2*TM))*
        ((D^2*p[1]/(4*p[2]*TM)-1)*exp(-D^2*p[1]/(4*p[2]*TM))/SIGMA
    return(J)
}

H=c(0.72, 0.49, 0.30, 0.20, 0.16, 0.12)
TM=c(5.0, 10.0, 20.0, 30.0, 40.0, 50.0)

### Fixed parameter values.
D=60
Q=50
### We'll use sigma=1cm.
SIGMA=0.01*rep(1,length(H))
### The unknown/estimated parameters are S=p(1) and T=p(2).
p0=c(0.001, 1.0)
### Solve the least squares problem with LM.
PEST = lmarq('fun','jac',p0,1.0e-12,100)

```

---

loadMAT

*Load a Matlab matfile*


---

## Description

Load a Matlab matfile, rename the internal parameters to get R-objects

## Usage

```
loadMAT(fn, pos=1)
```

## Arguments

fn	file name of MATfile
pos	integer, position in search path, default=1

## Details

Program reads in previously saved mat-files and extracts the data, and renames the variables to match the book.

## Value

Whatever is in the MATfile

## Note

Matfiles are created using the matlab2R routines

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

---

l_curve_corner	<i>L Curve Corner</i>
----------------	-----------------------

---

**Description**

Retrieve corner of L-curve

**Usage**

```
l_curve_corner(rho, eta, reg_param)
```

**Arguments**

rho	misfit
eta	model norm or seminorm
reg_param	regularization parameter

**Value**

reg_corner	the value of reg_param with maximum curvature
ireg_corner	the index of the value in reg_param with maximum curvature
kappa	the curvature for each reg_param

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
#### Vertical Seismic Profile example
set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );
```



```

BIGU = flipGSVD(littleU, dim(G), dim(L1) )

U1 = BIGU$U
V1 =BIGU$V
X1=BIGU$X
Lam1=BIGU$C
M1=BIGU$S

K1 = l_curve_tgsvd(U1,t,X1,Lam1,G,L1);

rho1 =K1$rho
eta1 =K1$eta
reg_param1 =K1$reg_param
m1s =K1$m

### % store where the corner is (from visual inspection)
vcorn = l_curve_corner(rho1, eta1, reg_param1)

ireg_corner1=vcorn$reg_corner
rho_corner1=rho1[ireg_corner1];
eta_corner1=eta1[ireg_corner1];
print(paste('1st order reg corner is: ',ireg_corner1));

plot(rho1,eta1,type="b", log="xy" , xlim=c(1e-4, 1e-2) , ylim=c(6e-6, 2e-4) ,
      xlab="Residual Norm ||Gm-d||_2", ylab="Solution Seminorm ||Lm||_2" );
points(rho_corner1, eta_corner1, col='red', cex=2 )

```

---

l\_curve\_tgsvd

*L curve tgsvd*


---

### Description

L curve parameters and models for truncated gsvd regularization.

### Usage

```
l_curve_tgsvd(U, d, X, Lam, G, L)
```

### Arguments

U	U, output of GSVD
d	output of GSVD
X	output of GSVD

Lam	output of GSVD
G	output of GSVD
L	output of GSVD

**Value**

List:

eta	the solution seminorm $\ Lm\ $
rho	the residual norm $\ Gm - d\ $
reg_param	corresponding regularization parameters
m	corresponding suite of models for truncated GSVD

**Author(s)**

Jonathan M. Lees&lt;jonathan.lees@unc.edu&gt;

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
#### Vertical Seismic Profile example
set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );

BIGU = flipGSVD(littleU, dim(G), dim(L1) )

U1 = BIGU$U
V1 = BIGU$V
X1 = BIGU$X
Lam1 = BIGU$C
M1 = BIGU$S

K1 = l_curve_tgsvd(U1,t,X1,Lam1,G,L1);

rho1 = K1$rho
```

```

eta1 =K1$eta
reg_param1 =K1$reg_param
m1s =K1$m

### % store where the corner is (from visual inspection)
ireg_corner1=8;
rho_corner1=rho1[ireg_corner1];
eta_corner1=eta1[ireg_corner1];
print(paste('1st order reg corner is: ',ireg_corner1));

plot(rho1,eta1,type="b", log="xy", xlim=c(1e-4, 1e-2) , ylim=c(6e-6, 2e-4) ,
      xlab="Residual Norm ||Gm-d||_2", ylab="Solution Seminorm ||Lm||_2" );

```

---

l\_curve\_tikh\_gsvd      *L-curve tikh gsvd*

---

### Description

L-curve tikh gsvd

### Usage

```
l_curve_tikh_gsvd(U, d, X, Lam, Mu, G, L, npoints, varargin = NULL)
```

### Arguments

U	from the gsvd
d	data vector for the problem $G*m=d$
X	from the gsvd
Lam	from the gsvd
Mu	from the gsvd
G	system matrix
L	roughening matrix
npoints	Number of points
varargin	alpha_min, alpha_max: if specified, constrain the logarithmically spaced regularization parameter range, otherwise an attempt is made to estimate them from the range of generalized singular values

### Details

Uses output of GSVD

**Value**

eta - the solution seminorm  $\|Lm\|$   
rho - the residual norm  $\|Gm - d\|$   
reg\_param - corresponding regularization parameters  
m - corresponding suite of models for truncated GSVD

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```
#### Vertical Seismic Profile example
set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );

BIGU = flipGSVD(littleU, dim(G), dim(L1) )

U1 = BIGU$U
V1 =BIGU$V
X1=BIGU$X
Lam1=BIGU$C
M1=BIGU$S

K1 = l_curve_tikh_gsvd(U1,t,X1,Lam1,M1, G,L1, 25);

rho1 =K1$rho
eta1 =K1$eta
reg_param1 =K1$reg_param
m1s =K1$m

### store where the corner is (from visual inspection)
ireg_corner1=8;
rho_corner1=rho1[ireg_corner1];
eta_corner1=eta1[ireg_corner1];
print(paste('1st order reg corner is: ',ireg_corner1));

plot(rho1,eta1,type="b", log="xy", xlim=c(1e-4, 1e-2) , ylim=c(6e-6, 2e-4) ,
      xlab="Residual Norm  $\|Gm-d\|_{-2}$ ", ylab="Solution Seminorm  $\|Lm\|_{-2}$ " );
```

---

l\_curve\_tikh\_svd      *L-curve Tikhonov*

---

**Description**

L-curve for Tikhonov regularization

**Usage**

```
l_curve_tikh_svd(U, s, d, npoints, varargin = NULL)
```

**Arguments**

U	matrix of data space basis vectors from the svd
s	vector of singular values
d	the data vector
npoints	the number of logarithmically spaced regularization parameters
varargin	alpha_min, alpha_max: if specified, constrain the logarithmically spaced regularization parameter range, otherwise an attempt is made to estimate them from the range of singular values

**Details**

Calculates the L-curve

**Value**

eta	the solution norm $\ m\ $ or seminorm $\ Lm\ $
rho	the residual norm $\ Gm - d\ $
reg_param	corresponding regularization parameters

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```

#### Vertical Seismic Profile example
set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );

BIGU = flipGSVD(littleU, dim(G), dim(L1) )

U1 = BIGU$U
V1 =BIGU$V
X1=BIGU$X
Lam1=BIGU$C
M1=BIGU$S

K1 = l_curve_tikh_svd(U1, diag(M1) , X1, 25, varargin = NULL)

rho1 =K1$rho
eta1 =K1$eta
reg_param1 =K1$reg_param
m1s =K1$m

### store where the corner is (from visual inspection)
ireg_corner1=8;
rho_corner1=rho1[ireg_corner1];
eta_corner1=eta1[ireg_corner1];
print(paste("1st order reg corner is: ",ireg_corner1));

plot(rho1,eta1,type="b", log="xy" ,
      xlab="Residual Norm ||Gm-d||_2", ylab="Solution Seminorm ||Lm||_2" );

```

---

mcmc

*Maximum likelihood Models*


---

**Description**

Maximum likelihood Models

**Usage**

```
mcmc(aologprior, aologlikelihood, agenerate, alogproposal, m0, niter)
```

**Arguments**

alogprior	Name of a function that computes the log of the prior distribution.
aloglikelihood	Name of a function the computes the log of the likelihood.
agenerate	Name of a function that generates a random model from the current model using the
alogproposal	Name of a function that computes the log of the proposal distribution $r(x,y)$ .
$m_0$	Initial model
niter	Number of iterations to perform

**Value**

mout	MCMC samples
mMAP	Best model found in the MCMC simulation.
accrate	Acceptance rate

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```

fun <-function(m,x)
{
  y=m[1]*exp(m[2]*x)+m[3]*x*exp(m[4]*x)
  return(y)
}

generate <-function( x) {
  y=x+step*rnorm(4)
  return(y)
}

logprior <-function(m)
{
  if( (m[1]>=0) & (m[1]<=2) &
      (m[2]>=-0.9) & (m[2]<=0) &
      (m[3]>=0) & (m[3]<=2) &
      (m[4]>=-0.9) & (m[4]<=0) )
  {
    lp=0
  }
  else
  {
    lp= -Inf
  }

  return(lp)
}
loglikelihood <-function(m)

```

```

{
  fvec=(y-fun(m,x))/sigma
  L=(-1/2)*sum(fvec^2)
  return(L)
}
logproposal <-function(x,y)
{
  LR=(-1/2)*sum((x-y)^2/step^2)
  return(LR)
}

### Generate the data set.
x=seq(from=1, by=0.25, to=7.0)

mtrue=c(1.0, -0.5, 1.0, -0.75)

ytrue=fun(mtrue,x)

sigma=0.01*rep(1, times= length(ytrue) )

y=ytrue+sigma*rnorm(length(ytrue) )

### set the MCMC parameters
### number of skips to reduce autocorrelation of models
skip=100
### burn-in steps
BURNIN=1000
### number of posterior distribution samples
N=4100
### MVN step size
step = 0.005*rep(1,times=4)

### We assume flat priors here
m0 = c(0.9003,
       -0.5377,
        0.2137,
       -0.0280)

alogprior='logprior'
aloglikelihood='loglikelihood'
agenerate='generate'
alogproposal='logproposal'

### ### initialize model at a random point on [-1,1]

### m0=(runif(4)-0.5)*2
### this is the matlab initialization:
m0 = c(0.9003,
       -0.5377,
        0.2137,
       -0.0280)

MM = mcmc('logprior','loglikelihood','generate','logproposal',m0,N)

```



```
mout = MM[[1]]  
mMAP= MM[[2]]  
pacc= MM[[3]]
```

---

Mnorm

*Matrix Norm*

---

### Description

Matrix Norm

### Usage

```
Mnorm(X, k = 2)
```

### Arguments

X	matrix
k	norm number

### Details

returns the largest singular value of the matrix or vector

### Value

Scalar Norm

### Note

if k=1, absolute value; k=2 2-norm (rms); k>2, largest singular value.

### Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

### Examples

```
x = runif(10)  
  
Mnorm(x, k = 2)
```

nnz

*Non-zeros*

---

**Description**

Number of non-zero elements in a vector

**Usage**

```
nnz(h)
```

**Arguments**

h                    vector

**Value**

integer

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```
zip<-rnorm(15)
nnz(zip)
```

---

occam*Occam inversion*

---

**Description**

Occam's inversion

**Usage**

```
occam(afun, ajac, L, d, m0, delta)
```

**Arguments**

afun	character, function handle that computes the forward problem
ajac	character, function handle that computes the Jacobian of the forward problem
L	regularization matrix
d	data that should be fit
m0	guess at the model
delta	cutoff to use for the discrepancy principle portion

**Value**

vector, model found

**Note**

This is a simple brute force way to do the line search. Much more sophisticated methods are available. Note: we've restricted the line search to the range from 1.0e-20 to 1. This seems to work well in practice, but might need to be adjusted for a particular problem.

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**See Also**

bayes

---

phi *Integral of Normal Distribution*

---

**Description**

normal distribution and returns the value of the integral

**Usage**

phi(x)

**Arguments**

x endpoint of integration (scalar)

**Value**

value of integral

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**See Also**

erf

**Examples**

```
x <- 1.0
##  pracma::erf(x)
phi(x)
phiinv( phi(x) )
```

---

phiinv

*Inverse Normal Distribution Integral*

---

**Description**

Calculates the inverse normal distribution from the value of the integral

**Usage**

```
phiinv(x)
```

**Arguments**

x                    endpoint value of integration (scalar)

**Value**

value of integral (scalar)

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**See Also**

phi

**Examples**

```
x <- 1.0
##  pracma::erf(x)
phi(x)
phiinv( phi(x) )
```

---

picard\_vals

*Picard plot*

---

**Description**

Picard plot parameters for subsequent plotting.

**Usage**

```
picard_vals(U, sm, d)
```

**Arguments**

U	the U matrix from the SVD or GSVD
sm	singular values in decreasing order, or the GSVD lambdas divided by the mus in decreasing order
d	data to fit, right hand side

**Details**

The Picard plot is a method of helping to determine regularization schemes.

**Value**

List:

utd	the columns of U transposed times d
utd_norm	utd./sm

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**See Also**

GSVD

**Examples**

```
####
n = 20
G = shawG(n,n)
spike = rep(0,n)
spike[10] = 1
dspiken = G

set.seed(2015)
dspiken = dspiken + 6e-6 *rnorm(length(dspiken))
Utube=svd(G);
U = Utube$u
V = Utube$v
S = Utube$d
s=Utube$d
R3 = picard_vals(U,s,dspiken);
utd = R3$utd
utd_norm= R3$utd_norm
#### Produce the Picard plot.

x_ind=1:length(s);
##
plot( range(x_ind) , range(c(s ,abs(utd),abs(utd_norm))),
      type='n', log='y', xlab="i", ylab="" )
lines(x_ind,s, col='black')
points(x_ind,abs(utd), pch=1, col='red')
points(x_ind,abs(utd_norm), pch=2, col='blue')

title("Picard Plot for Shaw Problem")
```

---

plotconst

*Plot constant model*


---

**Description**

Plots a model in piecewise constant form over  $n$  subintervals, where  $n$  is the length of  $x$ .

**Usage**

```
plotconst(x, l, r, ...)
```

**Arguments**

$x$	model to be plotted
$l$	left endpoint of plot

r                   right endpoint of plot  
...                  graphical parameters

**Details**

Used for plotting vector models

**Value**

graphical side effects

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**See Also**

linesconst

**Examples**

```
zip = runif(25)
plotconst(zip, 0, 1 )
linesconst(runif(25) , 0, 1 , col='red' )
```

---

quadlin

*Lagrange multiplier technique*

---

**Description**

Quadratic Linearization

**Usage**

```
quadlin(Q, A, b)
```

**Arguments**

Q                   positive definite symmetric matrix  
A                   matrix with linearly independent rows  
b                   data vector

**Details**

Solves the problem:  $\min (1/2) t(x)*Q*x$  with  $Ax = b$ . using the Lagrange multiplier technique, where  $Q$  is assumed to be symmetric and positive definite and the rows of  $A$  are linearly independent.

**Value**

list:

x                    vector of solution values  
lambda              Lagrange multiplier

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```
###%    Radius of the Earth (km)
      Re=6370.8;
rad = 5000
ri=rad/Re;

q=c(1.083221147, 1.757951474)
H = matrix(rep(0, 4), ncol=2, nrow=2)

H[1,1]=1.508616069 - 3.520104161*ri + 2.112062496*ri^2;
H[1,2]=3.173750352 - 7.140938293*ri + 4.080536168*ri^2;
H[2,1]=H[1,2];
H[2,2]=7.023621326 - 15.45196692*ri + 8.584426066*ri^2;
A1 =quadlin(H,t(q), 1.0 );
```

---

rnk	<i>Rank of Matrix</i>
-----	-----------------------

---

**Description**

Return the rank of a matrix. Not to be confused with the R function rank.

**Usage**

```
rnk(G, tol = 1e-14)
```

**Arguments**

G                    Matrix  
tol                  machine tolerance for small numbers



**Details**

Number of singular values greater than tol.

**Value**

integer, number of non-zero singular values

**Note**

duplicate the matlab function rank

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**See Also**

svd

**Examples**

```
hilbert <- function(n) { i <- 1:n; 1 / outer(i - 1, i, "+") }  
X <- hilbert(9)[,1:6]  
rnk(X)
```

---

setDesignG

*Set a Design Matrix.*

---

**Description**

Creata design matrix for simulating a tomographic inversion on a simple grid.

**Usage**

```
setDesignG()
```

**Details**

Set up a simple design matrix for tomographic in version. This is used in examples and illustrations of tomographics and matrix inversion methods.

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**Examples**

```
G = setDesignG()

### show the 56-th row
g = matrix( G[56,] , ncol=16, nrow=16)
imagesc(g)

## Not run:
### show total coverage
zim = matrix(0 , ncol=16, nrow=16)
for(i in 1:dim(G)[1])
{
g = matrix( G[i,] , ncol=16, nrow=16)
zim =zim + g
}
image(zim)

## End(Not run)
```

---

shawG

*Shaw Model of Slit Diffraction*

---

**Description**

Creates the design matrix for the Shaw inverse problem of diffraction through a narrow slot.

**Usage**

```
shawG(m, n)
```

**Arguments**

m	integer, number of rows
n	integer number of columns

**Details**

See Aster's book for a details explanation.

**Value**

Matrix used for creating data and inversion.

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

C. B. Shaw, Jr., "Improvements of the resolution of an instrument by numerical solution of an integral equation", J. Math. Anal. Appl. 37: 83-112, 1972.

**Examples**

```
n = 20
G = shawG(n,n)

spike = rep(0,n)
spike[10] = 1

dspiken = G %*% spike

plot(dspiken)
```

---

sirt

*SIRT Algorithm for sparse matrix inversion*

---

**Description**

Row action method for inversion of matrices, using SIRT algorithm.

**Usage**

```
sirt(A, b, tol, maxiter)
```

**Arguments**

A	Design Matrix
b	vector, Right hand side
tol	numeric, tolerance for stopping
maxiter	integer, Maximum iterations

**Details**

Iterates until conversion

**Value**

Solution vector

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

## References

Lees, J. M. and R. S. Crosson (1989): Tomographic inversion for three-dimensional velocity structure at Mount St. Helens using earthquake data, *J. Geophys. Res.*, 94(B5), 5716-5728.

## See Also

art, kac

## Examples

```
set.seed(2015)
G = setDesignG()
### Setup the true model.
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);

mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;
mtruem[10,9]=1; mtruem[10,11]=1;
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;
mtruem[2,3]=1; mtruem[2,4]=1;
mtruem[3,3]=1; mtruem[3,4]=1;

### reshape the true model to be a vector
mtruev=as.vector(mtruem);

### Compute the data.
dtrue=G %*% mtruev;

### Add the noise.
d=dtrue+0.01*rnorm(length(dtrue));

msirt<-sirt(G,d,0.01,200)
par(mfrow=c(1,2))
imagesc(matrix(mtruem,16,16) , asp=1 , main="True Model" );

imagesc(matrix(msirt,16,16) , asp=1 , main="SIRT Solution" );
```

---

tinv

*Inverse T-distribution*

---

## Description

Inverse T-distribution, qt

## Usage

tinv(p, nu)

**Arguments**

p	P-value
nu	degrees of freedom

**Details**

Wrapper for qt

**Value**

Quantile for T-distribution

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**See Also**

qt

**Examples**

```
tinvs(.4, 10)
```

---

USV

*Singular Value Decomposition*

---

**Description**

Singular Value Decomposition

**Usage**

```
USV(G)
```

**Arguments**

G	Matrix
---	--------

**Details**

returns matrices U, S, V according to matlab convention.

**Value**

list:

U	Matrix
S	Matrix, singular values
V	Matrix

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**See Also**

svd

**Examples**

```
hilbert <- function(n) { i <- 1:n; 1 / outer(i - 1, i, "+") }
X <- hilbert(9)[,1:6]

h = USV(X)

print( h$U )
```

---

Vnorm

*Vector 2-Norm*

---

**Description**

Vector 2-Norm.

**Usage**

Vnorm(X)

**Arguments**

X                    numeric vector

**Value**

Numeric scale norm

**Note**

This function is intended to duplicated the matlab function norm.

**Author(s)**

Jonathan M. Lees&lt;jonathan.lees@unc.edu&gt;

**Examples**

```
V = Vnorm(rnorm(10))
```

---

vspprofile

*Vertical Seismic Profile In 1D*


---

**Description**

Example vertical 1-dimensional seismic profile used for setting up examples for inverse theory.

**Usage**

```
vspprofile(M = 50, N = 50, maxdepth = 1000, deltobs = 20,
noise = 2e-04, M1 = c(9000, -6, 0.001))
```

**Arguments**

M	integer, number of rows in in design matrix G, default=50
N	integer, number of columns in design matrix G, default=50
maxdepth	Maximum depth of model, default = 1000
deltobs	integer, sampling interval in depth, default=20
noise	gaussian noise multiplier, default=2e-04
M1	3-vector, linear model for velocity versus depth model

**Details**

Vertical seismic profile in 1D dimension used for setting up examples in PEIP. Given a simple velocity profile, defined by input parameter M1 create the travel times and design matrix used for solving an inverse problem. The velocity model is defined as depth versus velocity, and the function inverts that from the slowness. Any model could be used to replace this model. The default model here is taken from an inversion in the Aster book.

**Value**

list:

G	M by N design matrix
tee	true travel times from model
t2	travel times with noise added

depth	depth samples of model
vee	velocity at the depths indicated
M	input M
N	input N
maxdepth	input maxdepth
deltobs	input delta observation
noise	input noise
M1	True model used for depth versus velocity

**Author(s)**

Jonathan M. Lees<jonathan.lees@unc.edu>

**References**

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

**Examples**

```
V = vspprofile()
### plot quadratic velocity profile
plot(V$vee, -V$depth, main="VSP: velocity increasing with depth")
dobs = seq(from=V$deltobs, to=V$maxdepth, by=V$deltobs)
### plotdepth versus time (not linear)
plot(dobs, V$t2)
abline(lm(V$t2 ~ dobs) )
```



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